Received: *September 13, 2000*, Accepted: *October 4, 2000*

Large marginal deformations in string field theory

Ashoke Sen
Mehta Research Institute of Mathematics and Mathematical Physics
Chhatnag Road, Jhoosi, Allahabad 211019, India
E-mail: asen@thwgs.cern.ch, sen@mri.ernet.in

Barton Zwiebach
Center for Theoretical Physics, Massachussetts Institute of Technology
Cambridge, MA 02139, USA
E-mail: zwiebach@mitlns.mit.edu

Abstract: We use the level truncation scheme to obtain accurate descriptions of open bosonic string field configurations corresponding to large marginal deformations such as background Wilson lines. To do so, we solve for all fields as functions of the massless string field, and confirm that the effective potential of the massless field becomes increasingly flat as the level of approximation is increased. Surprisingly, as a result of the merging of two branches of the solution — one originating at zero tachyon vev and the other originating at the tachyonic vacuum — this effective potential exists only for a finite range of values of the massless field. We use the D1 to D0-brane marginal transition on a circle to explore the possibility that this finite range corresponds to the infinite range of the conformal field theory parameter describing marginal deformations, but are unable to arrive at a definitive conclusion.

Keywords: Bosonic Strings, D-branes, Conformal Field Models in String Theory.

JHEP10(2000)009

Contents

1. Introduction and summary

It has been realised recently that string field theory (SFT) [1, 2, 3] provides a useful tool for studying the phenomenon of tachyon condensation in string theory [4]– [10] using the level truncation scheme developed in ref. [11]. This includes the open string tachyon on a D-brane of bosonic string theory, as well as on a non-BPS D-brane or a D-brane anti-D-brane pair in type II string theory. The analysis has been extended to discuss condensation of modes of the tachyon carrying momentum along compact directions as long as the effective $(\text{mass})^2$ of this mode remains negative [12, 13, 14]. In the language of two dimensional conformal field theory (CFT) describing the propagation of the string, switching on a vacuum expectation value (vev) for such tachyonic fields corresponds to switching on relevant perturbations. Although conformal field theory analysis has been used to gain insight into the phenomenon of tachyon condensation in some of these cases [15, 16], string field theory certainly seems to provide a unified approach to the study of these phenomena. An alternative approach to the study of these phenomena based on effective field theory on non-commutative spaces has been proposed in refs. [17]– [20], and its relationship to string field theory has been discussed in [21].

Typically D-branes also contain massless open string states. In many cases the potential for the fields associated with these modes has exact flat directions. Switching on a vev for these fields corresponds to exactly marginal deformations of the corresponding conformal field theory. A typical example of such a marginal deformation is the Wilson line; a constant vev of a U(1) gauge field along a compact direction. These deformations can be studied using well known techniques of conformal field theory. On the other hand, studying these in string field theory seems to be a difficult problem due to the following reason. Whereas we expect that the exact potential in string field theory will have an exact flat direction corresponding to each marginal deformation of the two dimensional conformal field theory, to any given order in the level truncation scheme the potential is not exactly flat. As a result, if we try to solve the equations of motion of string field theory using the level truncation scheme, then instead of getting a one parameter family of solutions corresponding to each marginal direction, we get isolated solutions.

This is the problem that we address in this paper. We take our marginal deformation parameter a_s, where the s stands for SFT, to be that associated with the constant vev of a U(1) gauge field. Instead of trying to solve the equations of motion of all components of the string field, we hold fixed a_s and solve for all other fields as a function of a_s using their equations of motion. This allows us to find the 'effective potential' for the massless field a_s. At any given order in the level expansion, this potential is not flat, and hence it has at most isolated extrema. We find, however, that as we increase the level of approximation, the potential becomes flatter, strongly suggesting that the exact effective potential is indeed flat. This shows that for a fixed vev of the massless field a_s, the solution of the equations of motion of all other fields gives an accurate representation of the string field configuration corresponding to the deformed conformal field theory. The flatness of the effective potential is consistent with the earlier result of Taylor [5] showing that the coefficient of the leading quartic term in the expansion of the effective potential around the origin does approach zero as the level of approximation is increased.

During the process of determining the effective potential of the marginal field, we find a surprise: the effective potential exists only if $|a_s|$ is less than a certain value $\overline{a}_s$. This restriction can be understood as follows. For zero vev of the marginal field, the equations of motion of the other fields have at least two solutions: the trivial solution where all other fields vanish, and the tachyonic vacuum solution studied in [11, 4, 6]. We shall refer to these two solutions as the M-branch, for marginal, and the V-branch for vacuum, respectively. When we switch on a small vev a_s of the marginal field, the two branches still remain, although the vev of the other fields associated with these two branches change slightly. For the study of the tachyonic vacuum, the V-branch is the relevant branch, but for our study, the M-branch is the relevant branch. We find that as we increase[1] the value of a_s these two branches come closer together, and at certain critical value of the vev, they meet. Beyond this point there are no real solutions associated to these branches. This phenomenon can be seen analytically at the level $(1,2)$ approximation to the potential, but we have checked numerically that this phenomenon persists at least up to the level $(4,8)$ approximation. Furthermore, the vev of the massless field a_s at which the two branches meet seems to converge rapidly to a finite value $\overline{a}_s$ as we increase the level of approximation. We shall refer to $\overline{a}_s$ as the critical value of the string field marginal parameter. This value, together with the values taken by the other fields in the theory define the *critical* string field.

This brings us to the question of interpretation of the critical value $\overline{a}_s$. Although this corresponds to a finite string field configuration, there is *a priori* no guarantee that it corresponds to a finite vev a_c (c for CFT parameter) of the gauge field which appears e.g. in the Born-Infeld action. There are now two possibilities:

1. the critical value $\overline{a}_s$ corresponds to $a_c = \infty$, or

2. the critical value $\overline{a}_s$ corresponds to a finite value $\overline{a}_c$ of the gauge field vev.

Unfortunately, with the information available at present, we are unable to distinguish between these two possibilities. If the first possibility holds, then this implies that an infinite distance in the CFT moduli space (as measured in Zamolodchikov metric) correponds to a finite shift in the string field. On the other hand, if the second possibility holds, then this would mean that open string field theory in a single coordinate system is unable to describe the full CFT moduli space. However, we can describe the full CFT moduli space by taking open string field theory in different coordinate patches.

To see that this can be done, note that all states are neutral under the gauge field and therefore the presence of the Wilson line does not affect the form of the correlation functions of the conformal field theory for a suitable choice of basis of states in the Hilbert space. Thus, the string field theory action has exactly the same form about any background Wilson line, and hence can span a given (finite) range of values of the Wilson line centered around the background value. Thus clearly the whole range of Wilson line vev can be spanned by putting together a set of string field theories formulated around different background values of the Wilson line. The fields in two such string field theories formulated around different background Wilson lines are presumably related by complicated non-linear field redefinitions. In the case of infinitesimal marginal deformations, these field redefinitions were worked out in ref. [22].

[1] Due to the existence of a Z_2 symmetry under which $a_s \to -a_s$, we can restrict our study to positive values of a_s only.

We are not only unable to decide between options (1) and (2) but it is also not clear to us whether or not the same phenomenon, i.e. the appearance of a critical value $\overline{a}_s$, also occurs in superstring field theory describing a single BPS D-brane configuration. In this case there is no analog of a non-trivial tachyonic vacuum from which another branch of the solution can originate, and therefore a critical value would have to arise by a different mechanism, possibly involving the massive string fields. Although there are tachyonic modes on a non-BPS D-brane or a brane-antibrane pair, the vev of the gauge field does not induce a tachyon vev due to the GSO Z_2 symmetry under which the tachyon is odd and the gauge field is even. In fact, no state in the GSO odd sector will acquire a vev. Hence the above comments for the BPS brane apply to these cases as well.

Although we carry out the analysis in the specific case of marginal deformations associated with the vev of a Wilson line, our results can be used in a more general context. First of all, instead of considering a gauge field vev along a direction tangential to the brane, we could consider giving a vev to the scalar field representing translation of a brane along a direction transverse to the brane. The effective potential for this mode will be identical to that of the Wilson line, and hence all our results apply to this mode. More generally, if we consider a situation where the bulk $c = 26$ matter conformal field theory has a U(1) current algebra, with the U(1) current satisfying either Dirichlet or Neumann boundary condition at the boundary of the world-sheet, then the U(1) current at the boundary of the world-sheet corresponds to a marginal operator, and our result can be used to describe the string field configuration corresponding to switching on this marginal deformation. In particular, this includes deformations which create a tachyonic lump on a circle of unit radius. As discussed in refs. [23, 24, 15, 16], if we consider a Dp-brane of bosonic string theory with one of its tangential directions x compactified on a circle of radius R, and switch on background tachyon field proportional to $\cos(x/R)$, the conformal field theory flows to that describing a D$(p-1)$-brane for $R \geq 1$. For $R = 1$ this describes a marginal deformation, since the tachyon vertex operator can be mapped to the boundary value of a U(1) current [24]. Hence the effective potential for this mode of the tachyon can be determined from our general results. We verify this explicitly by determining the tachyon potential to level $(4, 8)$ for arbitrary R close to but larger than one, and then showing that in the $R \to 1$ limit, this potential reduces to the level $(4, 8)$ potential involving the Wilson line. Using the results for the tachyon potential at level $(4, 8)$ approximation we also obtain a more accurate description of the lump solution for R close to but larger than one. This generalizes and extends the analysis of ref. [14].

The earlier analysis of [24] shows that for R close to but slightly larger than one, the effective tachyon potential has a minimum at $a_c = \pm 1/2\sqrt{2}$, corresponding to turning the Dp-brane into a D$(p-1)$-brane.[2] We use our present string field theory

[2] a_c is normalized such that it multiplies a vertex operator of unit norm in the CFT action.

analysis of the tachyon potential for R near one to attempt to find the expectation value of the string field theory variable a_s representing the same minimum. This gives the values of a_s corresponding to $a_c = \pm 1/2\sqrt{2}$. We use this to try to gain more insight into the functional relationship between a_c and a_s. Unfortunately this analysis does not quite answer the question as to whether $\overline{a}_s$ corresponds to finite or infinite value of a_c.

The rest of the paper is organised as follows. In section 2 we construct the relevant part of the string field theory action needed for computing the effective potential for the Wilson line, and use this to compute the effective potential. The computation is done analytically at level $(1,2)$ approximation and numerically up to level $(4,8)$ approximation. In section 3 we construct the string field theory action relevant for studying the lump solution on a circle of radius R, and discuss its equivalence with the action of section 2 for $R = 1$. We also use this potential for $R > 1$ to estimate the value of a_s for $a_c = 1/2\sqrt{2}$. This is used in section 4 to discuss the possible functional relation between a_s and a_c, in particular whether the upper limit on a_s corresponds to finite or infinite value of a_c. We conclude in section 5 with some comments. Appendices A and B contains the details of the string field theory action relevant for the analysis of sections 2 and 3 respectively.

2. String fields for Wilson line marginal deformations

In this section we begin our analysis of string fields corresponding to CFT marginal deformations. The setup is that of bosonic open string field theory describing the dynamics of a Dp-brane. We single out a particular coordinate x along the world volume of the brane and consider giving expectation value to the constant mode of the gauge field component A_x. This represents a marginal deformation of the BCFT describing the D-brane. Our aim is to find the string field corresponding to such deformations. We will not assume that the deformation is small. For our present analysis it will make no difference whether or not the x-direction is compact; we give x-independent expectation values to all fields, so that modes carrying non-zero momentum along x (or any other direction) are set to zero.

We begin by examining the lowest level approximation to the problem where happily, many of the features of the problem are already apparent. Then we discuss its generalization to higher levels.

2.1 Lowest level analysis: level $(1,2)$

At the lowest level (level $(1,2)$) approximation we must include the tachyon and the gauge field. The string field is therefore

$$|\Phi^{(1)}\rangle = (t_0 + a_s\, \alpha^X_{-1})\, c_1|0\rangle\,, \tag{2.1}$$

where t_0 denotes the level zero tachyon zero mode and a_s denotes the level one gauge field zero mode. α_n^X denotes the nth oscillator mode of X. We now evaluate the string field action to get the potential $\mathcal{V}(\Phi)(\equiv -S(\Phi)/(2\pi^2\mathcal{T}_p)$ where $\mathcal{T}_p$ is the tension of the D-brane) associated to this string field. A small calculation gives[3]

$$\mathcal{V}(t_0, a_s) = -\frac{1}{2}t_0^2 + \frac{1}{3}K^3 t_0^3 + K t_0 a_s^2 \,, \tag{2.2}$$

where $K = 3\sqrt{3}/4$. We note the absence of a quadratic and cubic term for a_s. This, of course, is expected by the standard CFT constraints on marginal operators. Note that the sign of the last term is the same as that of the second term. This will play a role in what follows, and is a result of the positive norm of the state $\alpha_{-1}^X|0\rangle$.

We now find the effective potential for a_s by integrating out classically the tachyon field t_0. Since the equation of motion is quadratic we find two solutions

$$t_0^M = \frac{4}{81\sqrt{3}}\Big(-\sqrt{64 - 729a_s^2} + 8\Big), \qquad t_0^V = \frac{4}{81\sqrt{3}}\Big(\sqrt{64 - 729a_s^2} + 8\Big). \tag{2.3}$$

These are the marginal M-branch solution and the vacuum V-branch solution respectively. Indeed one can see that for $a_s = 0$ we get $t_0^M = 0$, which is the value for the tachyon at the maximum of the potential, while t_0^V is the familiar tachyon expectation value at the local minimum of the cubic potential. It is clear from the above equations that there are no real solutions for t_0 unless

$$|a_s| \leq \frac{8}{27} \equiv \overline{a}_s^{(1,2)} \,. \tag{2.4}$$

Note that at this point the two branches for t_0 meet.

It is of interest to understand the nature of the effective potential for a_s. Substituting the values of t_0 into $\mathcal{V}(t_0, a_s)$ and letting $\mathcal{V}^{M/V}(a_s) \equiv \mathcal{V}(t_0^{M/V}(a_s), a_s)$ we obtain:

$$\mathcal{V}^{M/V}(a_s) = \frac{2}{59049}\Big(-512 + 8748a_s^2 \pm (64 - 729a_s^2)^{3/2}\Big) \,, \tag{2.5}$$

where the top sign (the $+$) goes for the M-branch, and the bottom sign is for the V-branch. We are particularly interested in the M-branch, where we expect to have a_s represent a marginal direction. Thus ideally $\mathcal{V}^M(a_s)$ should have been identically zero. While it is not zero, the function is certainly relatively flat. It is a monotonically increasing function, and $2\pi^2\mathcal{T}_p\mathcal{V}^M(\overline{a}_s^{(1,2)}) = 0.17\,\mathcal{T}_p$, indicating that at the end of the domain of definition the potential energy for the marginal direction fails to be zero by about 17% of the D-brane tension. We can expand $\mathcal{V}^M(a_s)$ for small a_s finding:

$$\mathcal{V}^M(a_s) = \frac{27}{32}a_s^4 + \frac{6561}{4096}a_s^6 + \cdots \,. \tag{2.6}$$

[3] One can use, for example, the conservation method of ref. [25] to find the $t_0 a_s^2$ coupling. Using equation (4.18) one finds that $\langle c_1, \alpha_{-1}^X c_1, \alpha_{-1}^X c_1\rangle = \frac{16}{27}\langle c_1, c_1, c_1\rangle = \frac{16}{27}K^3 = K$.

JHEP10(2000)009

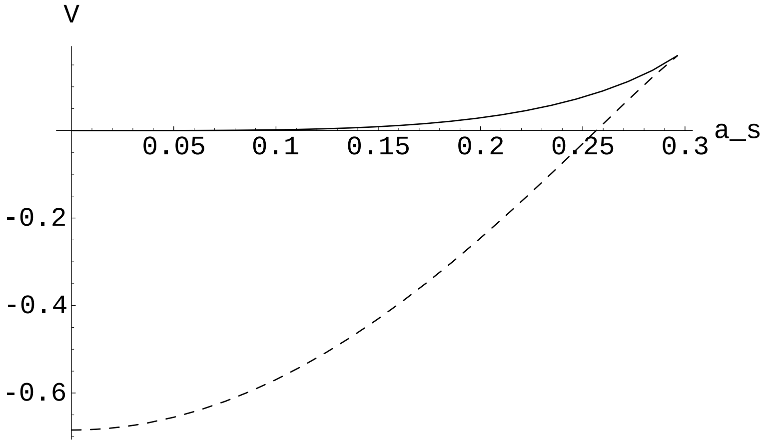

Figure 1: The level $(1,2)$ effective potential $V \equiv (2\pi^2 \mathcal{V}^{M/V}(a_s))$ for a_s on the M-branch (solid line) and V-branch (dashed line). The two branches meet at $a_s^{(1,2)} = 8/27$.

We shall see that as the level of approximation is increased the potential $\mathcal{V}^M$ becomes flatter. The leading coefficient (as well as the other expansion coefficients) will become smaller. The computation of the leading quartic term in the above equation is equivalent to the computation of the quartic gauge field interaction in ref. [5].

We can also examine the V-branch for a_s small. We get

$$\mathcal{V}^V(a_s) = -\frac{2048}{59049} + \frac{16}{27} a_s^2 + \cdots . \tag{2.7}$$

Notice the leading constant term. The numerical value of $2\pi^2 \mathcal{T}_p \mathcal{V}^V(a_s = 0)$ is $-.68\mathcal{T}_p$. This is 68% of the energy density of the original brane, and approaches 100% of the brane tension as we increase the level of approximation [4, 6]. Note also that the quadratic term for a_s does not vanish (and will not become smaller as the level of approximation is increased). This is consistent with the expectation that there are no massless states around the stable vacuum. The marginal direction has been lifted.[4]

2.2 Higher level analysis: up to level $(4, 8)$

We gave before the fields necessary to compute the effective potential of the marginal parameter to level $(1,2)$. This included the tachyon and the level one field $\alpha^X_{-1} c_1 |0\rangle$ (see (3.1)). Note that the latter state is odd under both the twist symmetry (the twist eigenvalue is $\Omega = (-)^l$, with l the level), and the $X \to -X$ transformation. Thus it is even under the combined operation of the twist and parity transformation. Since this is a symmetry of the string field theory action [27], it is clear that we can get a consistent solution of the full string field theory equations of motion by setting to zero fields which are odd under this combined operation. Thus in extending our

[4] We thank W. Taylor for raising the question of the fate of this marginal direction on the stable vacuum, and for comparing with us his results to be published independently [26].

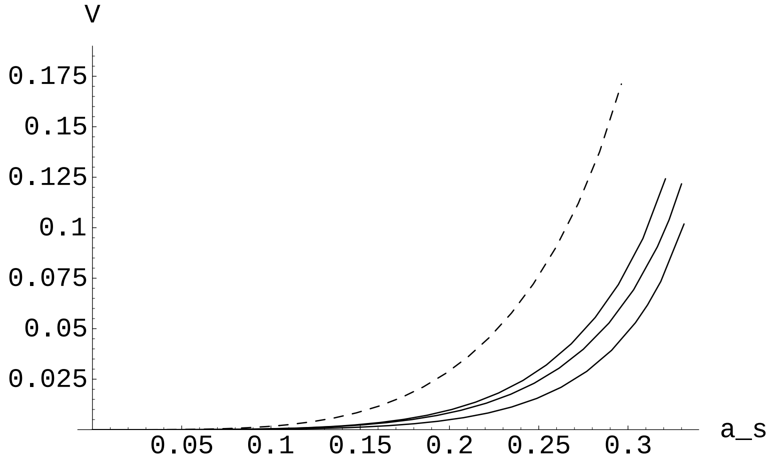

Figure 2: The various effective potentials $V \equiv (2\pi^2 \mathcal{V}^M(a_s))$ for the marginal parameter a_s on the M-branch. The dashed curve represents the level $(1,2)$ approximation, the successively flatter curves represent the level $(2,4)$, $(3,6)$ and $(4,8)$ approximations. Note that each potential has a different domain of definition.

analysis to higher level, we only need to include string field configurations which are even under the combined operation of the twist and the $X \to -X$ transformation. We shall further restrict the string field configuration by including only the zero momentum modes, using the Siegel gauge condition, and considerations involving closure of the $*$-product algebra of a subset of a string fields along the lines discussed in [28, 4, 14]. This amounts to including states of ghost number one, obtained by acting on $c_1|0\rangle$ with b_{-n}, c_{-n}, α^X_{-n} and L'_{-n} for $n > 0$. Following the notation used in [14], we denote by L^X_n the Virasoro generators associated with the world sheet field X and by L'_n the Virasoro generators associated with the other matter fields in the boundary CFT. To the extent possible, we shall try to label the states in terms of L^X_{-n} instead of α^X_{-n}.

We can now easily construct the list of string fields appearing at the next few levels. Let us begin with level two fields. Since these are automatically twist even, they must also be even under $X \to -X$. This means that we can get the usual (Virasoro and ghost-current) descendents of the tachyon field but cannot get descendents of the marginal field, since such descendents would be odd under $X \to -X$. As there is no new primary state in the CFT involving X at this level, the list of level 2 states in the string field is

$$|\Phi^{(2)}\rangle = \left(u_0 c_{-1} b_{-1} + v_0 L^X_{-2} + w_0 L'_{-2}\right) c_1|0\rangle\,. \tag{2.8}$$

Let us now continue to level three. Since all fields here are twist odd, they must also be odd under $X \to -X$. We can therefore allow all states obtained as ghost-current or Virasoro descendents of the level one state $\alpha^X_{-1} c_1|0\rangle$. One readily verifies

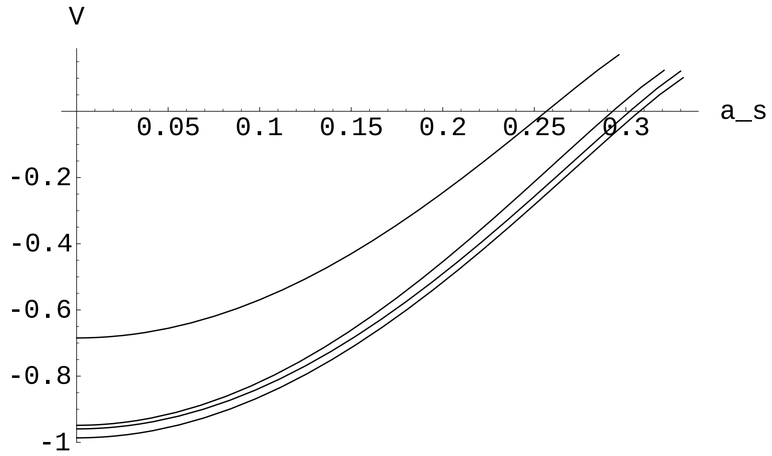

Figure 3: The various effective potentials $V \equiv (2\pi^2 \mathcal{V}^V(a_s))$ for the marginal parameter a_s on the V-branch. The top curve represents the level $(1,2)$ approximation, the successively lower curves represent the level $(2,4)$, $(3,6)$ and $(4,8)$ approximations.

that this exhausts the list of possible states,[5] and gives a total of four level three fields:

$$|\Phi^{(3)}\rangle = \left(sc_{-1}b_{-1} + rL^X_{-2} + \bar{r}L'_{-2} + yL^X_{-1}L^X_{-1}\right)|\varphi_a\rangle\,, \qquad |\varphi_a\rangle = \alpha^X_{-1}c_1|0\rangle\,. \tag{2.9}$$

Finally we proceed to the list of fields at level four. Being twist even, all states that are ghost current or Virasoro descendents of the tachyon must be included. These give a total of ten states. There is one more twist even state at this level, — a level four primary of CFT(X)

$$|p_4\rangle = \left(\alpha^X_{-3}\alpha^X_{-1} - \frac{3}{4}(\alpha^X_{-2})^2 - \frac{1}{2}(\alpha^X_{-1})^4\right)c_1|0\rangle\,. \tag{2.10}$$

Therefore the level four string field is given as

$$\begin{aligned}|\Phi^{(4)}\rangle = g|p_4\rangle + \Big(&aL^X_{-4} + \bar{a}L'_{-4} + bL^X_{-2}L^X_{-2} + \bar{b}L'_{-2}L'_{-2} + \widehat{b}L'_{-2}L^X_{-2} + \\ &+ c\,c_{-3}b_{-1} + d\,b_{-3}c_{-1} + e\,b_{-2}c_{-2} + (f\,L^X_{-2} + \bar{f}L'_{-2})c_{-1}b_{-1}\Big)c_1|0\rangle\,.\end{aligned} \tag{2.11}$$

We can now compute the potential $\mathcal{V}$ at various levels of approximation by standard procedure. The results of this computation are given in appendix A. At each level, we can determine the effective potential for the Wilson line in the M-branch by (numerically) eliminating all the other fields by their equations of motion. These results have been shown[6] in figure 2. We see from this figure that the effective potential becomes flatter as we increase the level of approximation. This can also

[5] One can count this as the number of level 3 ghost number 1 states built with α^X_{-n}, b_{-n}, c_{-n} and L'_{-n} oscillators which must have an odd number of α^X_{-n} oscillators.

[6] Due to the $a_s \to -a_s$ symmetry of the potentials, we have displayed them only for positive a_s.

JHEP10(2000)009

Level	$\overline{a}_s$	α_4^M	α_2^V	$2\pi^2\mathcal{V}^M(8/27)$	$2\pi^2\mathcal{V}^M(\overline{a}_s)$
$(1,2)$	0.296296	0.843752	0.592593	0.1712	0.1712
$(2,4)$	0.321374	0.200234	0.672892	0.0743	0.1254
$(3,6)$	0.330107	0.200234	0.631329	0.0605	0.1221
$(4,8)$	0.331428	0.096999	0.633432	0.0444	0.1020

Table 1: We show the variation of various quatities as a function of the level of the calculation. Here $\overline{a}_s$ denotes the maximal value possible for the string field marginal parameter. The coefficient α_4^M defines the leading quartic term in a_s in the effective potential on the M-branch. The coefficient α_2^V defines the leading quadratic term in a_s in the effective potential on the V-branch. We also show the value of the potential, normalized in units of the tension of the brane, for the maximal value ($\overline{a}_s$) of $|a_s|$ at level $(1,2)$, and for the end of the range at each level.

be verified by computing the coefficient α_4^M of the a_s^4 term in the expression for the effective potential (see, for example, (2.6)), which has been listed in table 1 and is seen to decrease as we increase the level of approximation. These results for α_4^M are in agreement with those of ref. [5]. In each case we find a maximum value $\overline{a}_s$ of $|a_s|$ beyond which the effective potential ceases to exist. These values have also been listed in table 1 and are seen to converge rapidly to about 0.33. The same procedure can be carried out to determine the effective potential in the V-branch (by choosing different initial data for obtaining the solution). These results have been shown in figure3. As we see from this figure, the effective potential on the V-branch does not become flat as we increase the level of approximation. A quantitative measure of this is the coefficient α_2^V of the a_s^2 term in the potential (see, for example, (2.7)); as seen from table 1, it converges to a finite value as we increase the level of approximation.

3. Tachyonic lump solution near marginality

The setup here is that of ref. [14], namely, we consider a D-brane with one of its spatial dimensions wrapped around a circle of radius R. The object of interest is the potential for string field modes that are either space-time constants or carry momentum along this circle. This problem was indeed analysed in ref. [14] for various values of $R > 1$. Our interest here, however, is the $R \to 1$ limit where the tachyonic mode t_1 carrying unit momentum along the circle becomes marginal. Giving a vev to this mode can be shown to be equivalent to giving a vev to a Wilson line [24]. Thus we expect this string field tachyon condensation problem to be related to the string field Wilson line problem discussed in the previous section. As we shall see by direct examination of the corresponding string field potentials, this is indeed the case. We shall also be able to gain some additional information about the Wilson line problem by exploiting this equivalence.

JHEP10(2000)009

In the same spirit as for the case of the marginal field, we introduce radius dependent effective potentials $\widetilde{\mathcal{V}}(t_1;R)$ for t_1. We take the full string field potential to a given approximation, and for fixed values of R and t_1 eliminate all other variables by using their equations of motion. Choosing between the marginal[7] or vacuum solution branches, this defines, for the fixed chosen value of R, the effective potentials $\widetilde{\mathcal{V}}^M(t_1;R)$ and $\widetilde{\mathcal{V}}^V(t_1;R)$ for t_1. Except for the lowest level approximation, the effective potentials are calculated only numerically, using the analytic expressions for the full string field potential.

We also attempt here to estimate the vev of t_1 which at the radius $R=1$ leads to the formation of the lump. Via the identification $t_1 \leftrightarrow \sqrt{2}a_s$ to be established below, we obtain an estimate for the vev of the string field parameter a_s leading to the formation of the lump. On the other hand, we can describe the formation of the lump at $R=1$ as a marginal deformation of the boundary CFT. Let us denote by a_c the parameter labelling this marginal deformation, normalised so that in the action of the deformed CFT, it multiplies $\int dx\varphi(x)$, with the marginal operator φ normalized so that $\langle\varphi|\varphi\rangle = 1$. Then the value of a_c, which corresponds to the formation of the lump solution at $R=1$, is $a_c = \pm 1/2\sqrt{2}$ [23, 24]. These pieces of information will be used in section 4 to investigate the functional relationship between a_s and a_c.

3.1 Tachyon potential for D-brane on a circle at level $(1,2)$

Since we will be particularly interested in the case $R \to 1$ where the first tachyon harmonic t_1 becomes marginal, we will measure level as if the radius equals one. As in ref. [14], we shall consider string field configurations which are even under twist, *and* the $X \to -X$ transformation. At level one the string field now includes the zero mode t_0 of the tachyon and its first harmonic t_1:

$$|\widetilde{\Phi}^{(1)}\rangle = \left(t_0 + t_1 \cos\left(\frac{X(0)}{R}\right)\right) c_1 |0\rangle\,. \tag{3.1}$$

The potential, given in [14, eqs. (3.5) and (3.6)] is

$$\widetilde{\mathcal{V}}(t_0,t_1;R) = -\frac{1}{2}t_0^2 + \frac{1}{3}K^3 t_0^3 - \frac{1}{4}\left(1-\frac{1}{R^2}\right)t_1^2 + \frac{1}{2}K^{3-2/R^2} t_0 t_1^2\,. \tag{3.2}$$

When $R=1$ the potential simplifies to

$$\widetilde{\mathcal{V}}(t_0,t_1;R=1) = -\frac{1}{2}t_0^2 + \frac{1}{3}K^3 t_0^3 + \frac{1}{2}\,K t_0 t_1^2\,, \tag{3.3}$$

where t_1 now clearly represents a massless state. Comparing this with (2.2) we see that the two potentials agree if we identify

$$t_1 \leftrightarrow \sqrt{2}a_s\,. \tag{3.4}$$

[7]Note that the deformation associated with t_1 is no longer marginal for $R \neq 1$. But we shall continue to refer to this branch as the marginal branch.

This factor of $\sqrt{2}$ is simply a reflection of the fact that the state $\cos(X(0)/R)|0\rangle$ has norm $(1/\sqrt{2})$, whereas $\alpha^X_{-1}|0\rangle$ has unit norm. As we shall see in the next subsection, this identification will be possible to implement to higher level, as expected from the CFT argument [24].

The tachyon case, however, lends itself to an interesting analysis for R slightly above one. Again, we can integrate t_0 using its field equation to find an effective potential for the field t_1. The quadratic equation for t_0 only has real solutions when

$$|t_1| \leq \bar{t}_1(R) \equiv \frac{8\sqrt{2}}{27}\, K^{-1+1/R^2}\,. \tag{3.5}$$

Note that the maximum possible value $\bar{t}_1$ of t_1 becomes larger as the radius is decreased towards the value $R = 1$. The critical value $\bar{t}_1$ at $R = 1$, as required, is in agreement with (2.4) given the identification (3.4). The resulting M-branch effective potential $\widetilde{\mathcal{V}}^M(t_1; R)$ for the t_1 field is a bit complicated and the explicit form is not very illuminating, but it has a local minimum at:

$$t_1 = \pm K^{-3+2/R^2} \sqrt{1-\frac{1}{R^2}}\,\sqrt{1-\frac{1}{2}K^{2/R^2}\left(1-\frac{1}{R^2}\right)} \equiv \pm t_1^{(0)}(R)\,. \tag{3.6}$$

This describes the one lump solution for a given value of R. This value of $t_1^{(0)}(R)$, of course, agrees with the analysis of ref. [14]. Here we would like to use $t_1^{(0)}(R)$ to estimate the vev of t_1 leading to the one lump solution at $R = 1$, namely, at marginality. The $R \to 1$ limit of $t_1^{(0)}(R)$, however, simply vanishes! The source of this problem can be traced to the following facts. The effective potential for t_1 has two parts: the term quadratic in t_1 which vanishes identically at $R = 1$, and the higher order terms which do not vanish identically at a given level of approximation, but are expected to vanish when we compute the potential exactly. Due to this, at *any* given level of approximation the minimum of $\widetilde{\mathcal{V}}^M(t_1; R)$ will approach the $t_1 = 0$ point as $R \to 1$. We shall try to get around this problem by working at a fixed value of R close to 1 (e.g. $\sqrt{1.1}$), and then increasing the level of approximation to determine the point that $t_1^{(0)}$ approaches for this fixed value of R.

3.2 Tachyon potential for D-brane on a circle at higher level

We now include fields at higher levels. Keeping only states which are even under twist and the $X \to -X$ transformations as in [14], we get the following set of states at level two:

$$|\widetilde{\Phi}^{(2)}\rangle = \Big(u_0 c_{-1} b_{-1} + v_0 L^X_{-2} + w_0 L'_{-2}\Big)\, c_1|0\rangle\,, \tag{3.7}$$

Note that this is identical to the list of states we generated in (2.8) for the Wilson line problem.

At level three we have,

$$|\widetilde{\Phi}^{(3)}\rangle = \left(u_1 c_{-1}b_{-1} + v_1 L^X_{-2} + w_1 L'_{-2} + z_1 L^X_{-1}L^X_{-1}\right)|\varphi_t\rangle\,,$$
$$|\varphi_t\rangle = \cos\Big(\frac{X(0)}{R}\Big)c_1|0\rangle\,. \tag{3.8}$$

At level four the fields to be included are:

$$|\widetilde{\Phi}^{(4)}\rangle = \widetilde{g}|p_4\rangle + t_2|\chi\rangle + \Big(aL^X_{-4} + \bar{a}L'_{-4} + bL^X_{-2}L^X_{-2} + \bar{b}L'_{-2}L'_{-2} + \widehat{b}L'_{-2}L^X_{-2} + cc_{-3}b_{-1} + $$
$$+ d\,b_{-3}c_{-1} + e\,b_{-2}c_{-2} + (fL^X_{-2} + \bar{f}L'_{-2})c_{-1}b_{-1}\Big)c_1|0\rangle\,, \tag{3.9}$$

where $|p_4\rangle$ has been defined in eq. (2.10), and

$$|\chi\rangle = \cos\left(\frac{2X(0)}{R}\right)|0\rangle\,. \tag{3.10}$$

The string field theory potential involving these fields has been computed and given in appendix B.

3.3 Relating marginal tachyons to Wilson lines

As we have mentioned several times, at $R = 1$ the CFT describing the D-brane marginal tachyon dynamics can be mapped to a Wilson line CFT problem. We have already seen that at level $(1,2)$ this identification is indeed realized by setting $t_1 \leftrightarrow \sqrt{2}a_s$ as indicated in (3.4). We shall now explain how this identification can be extended to show that the $R = 1$ tachyon problem is completely isomorphic to the Wilson line problem to level $(4,8)$. It follows from this that the effective potential $\mathcal{V}^M(a_s)$ for the Wilson line parameter and $\widetilde{\mathcal{V}}^M(t_1; R)$ for the first tachyon harmonic are related as

$$\mathcal{V}^M(a_s) = \widetilde{\mathcal{V}}^M\left(t_1 = \sqrt{2}a_s; R = 1\right). \tag{3.11}$$

We begin by arguing equivalence of the types of terms present in the complete string field potentials. We shall denote by $\mathcal{V}^{(M,N)}$ the level (M,N) approximation to $\mathcal{V}(\Phi)$, by $\mathcal{V}_{mm}$ the quadratic term in the potential for level m fields, and by $\mathcal{V}_{mnp}$ the cubic term in the potential coupling a level m, a level n and a level p field. As we did before, the potential terms $\widetilde{\mathcal{V}}$ for the tachyon lump case are distinguished from the Wilson line potential terms by the tilde. We now claim that the total level of all the terms entering a term in the potential must be even. For the case of the tachyon lump this follows readily by invariance of the string field action under the translation $X \to X + \pi R$; the odd level fields carry odd unit of momentum and hence are odd under this transformation whereas even level states carry even unit of momentum and so are even under this transformation. For the case of the Wilson line this follows from $X \to -X$ symmetry under which the odd level fields are odd, and the even level fields are even. These symmetries are eventually responsible for the $t_1 \to -t_1$ and $a_s \to -a_s$ symmetries of the corresponding effective potentials.

We list below, for convenience, the terms that appear in the string field potential relevant for the study of the Wilson line when we include fields up to a given level (while keeping the total level below 8):

$$\begin{aligned} \text{level zero}: &\ \mathcal{V}_{00}\,,\mathcal{V}_{000} \\ \text{level one}: &\ \mathcal{V}_{11}\,,\ \mathcal{V}_{011} \\ \text{level two}: &\ \mathcal{V}_{22}\,,\ \mathcal{V}_{002}\,,\ \mathcal{V}_{112}\,,\mathcal{V}_{022}\,,\ \mathcal{V}_{222}\,, \\ \text{level three}: &\ \mathcal{V}_{33}\,,\ \mathcal{V}_{013}\,,\ \mathcal{V}_{033}\,,\ \mathcal{V}_{123}\,,\ \mathcal{V}_{233} \\ \text{level four}: &\ \mathcal{V}_{44}\,,\ \mathcal{V}_{004}\,,\ \mathcal{V}_{114}\,,\ \mathcal{V}_{024}\,,\ \mathcal{V}_{224}\,,\ \mathcal{V}_{044}\,,\ \mathcal{V}_{134} \end{aligned} \tag{3.12}$$

For each level ℓ the list of terms up to and including those in the appropriate line give all interactions involving fields with level less than or equal to ℓ. A similar list with $\mathcal{V}$ replaced by $\widetilde{\mathcal{V}}$ appears for the string field potential relevant for the study of the lump solution on a circle.

With the identification $t_1 \leftrightarrow \sqrt{2}a_s$ we have already guaranteed that the terms listed in the first two lines above give exactly the same potentials as their tilde versions. Indeed, with φ_a and φ_t used to denote the Wilson and tachyon unit momentum marginal operators (see (2.9) and (3.8)) the agreement is the result of the following equality of correlators involving the Virasoro and ghost current primaries φ_a and φ_t:

$$\left\langle \frac{\varphi_a}{\sqrt{2}}, \frac{\varphi_a}{\sqrt{2}}, t \right\rangle = \langle \varphi_t, \varphi_t, t \rangle \,. \tag{3.13}$$

In this equation t denotes the CFT vertex operator for the zero-momentum tachyon $c_1|0\rangle$.

Let us now consider the third line in (3.12), that is include the new terms that involve level two fields (and fields with lower level). Note again that the level two fields in both cases (eqns. (2.8) and (3.7)) are identical and in fact we have used the same labels for them. It follows that all couplings involving level zero and level two fields in both string field potentials are identical. The only question is whether the $\mathcal{V}_{112}$ terms are the same as the $\widetilde{\mathcal{V}}_{112}$ terms. In other words, does the equality

$$\left\langle \frac{\varphi_a}{\sqrt{2}}, \frac{\varphi_a}{\sqrt{2}}, \Phi^{(2)} \right\rangle = \left\langle \varphi_t\,, \varphi_t\,, \widetilde{\Phi}^{(2)} \right\rangle \tag{3.14}$$

hold for any of the three level two fields in $\Phi^{(2)}$ and $\widetilde{\Phi}^{(2)}$? It does. Indeed, all fields in $\Phi^{(2)}$ ($\widetilde{\Phi}^{(2)}$) are either Virasoro or U(1) ghost-current descendents of the level zero tachyon. Since both φ_a and φ_t have the same dimension and ghost number, the equality (3.14) follows from (3.13).

Let us now consider the fourth line in (3.12), that is include the new terms that involve level three fields (and fields with lower level). Note the complete isomorphism manifest from equations (2.9) and (3.8): all these fields are just Virasoro and ghost-current descendents of φ_a and φ_t respectively. It follows from the normalizations of φ_a and φ_t that $\mathcal{V}_{33}$ and $\widetilde{\mathcal{V}}_{33}$ terms will match if we identify

$$s = \frac{u_1}{\sqrt{2}}\,, \qquad r = \frac{v_1}{\sqrt{2}}\,, \qquad \bar{r} = \frac{w_1}{\sqrt{2}}\,, \qquad y = \frac{z_1}{\sqrt{2}}\,. \tag{3.15}$$

Our remarks about descendents imply that the equality of correlators defining $\mathcal{V}_{013}$, $\mathcal{V}_{033}$, $\mathcal{V}_{123}$, and $\mathcal{V}_{233}$, with their tilde counterparts simply follow from (3.13).

Finally, let us consider the fifth and final line in (3.12), giving the new terms that involve level four fields (and fields with lower level). We do this in two stages. Let us first consider the common list of fields in (2.11) and (3.9). These ten fields, denoted by the same labels, are all Virasoro and ghost current descendents of the zero momentum tachyon. By the earlier arguments, all correlators involving these level four fields and fields of level ≤ 3 will agree in the Wilson and tachyon string field potentials. Now consider the remaining fields g (multiplying $|p_4\rangle$) in the Wilson case, and, $\widetilde{g}$ and t_2 (multiplying $|p_4\rangle$ and $|\chi\rangle$ respectively) in the tachyon lump case. All of these states are primaries. We now show that out of the two level four primaries in the tachyon lump case, only one linear combination needs to be kept at $R = 1$.

For this purpose consider in the tachyon lump case the complete list of primaries at all levels less than or equal to four appearing in the study of $\widetilde{\mathcal{V}}$:

$$\{\, t\,,\, \varphi_t\,,\, p_4\,,\, \chi \,\} \tag{3.16}$$

We now split them as follows

$$\left\{\, t\,,\, \varphi_t\,,\, \widetilde{p}_4 \equiv \frac{1}{2}(-p_4 + 9\chi) \right\}\,, \qquad d \equiv \frac{1}{2}(p_4 + 3\chi)\,. \tag{3.17}$$

In the first set we have included three vertex operators, and we have separated out a fourth one called d (d stands for decoupled). We now claim that the kinetic terms in the string field theory do not mix d with any operators in the first set. For the first two operators this is trivially so, and for the third it follows from $\langle p_4|c_0L_0|p_4\rangle = 81/2$, $\langle\chi|c_0L_0|\chi\rangle = 3/2$, and $\langle p_4|c_0L_0|\chi\rangle = 0$. Moreover, $\widetilde{p}_4$ has been normalized such that $\langle\widetilde{p}_4|c_0L_0|\widetilde{p}_4\rangle = \langle p_4|c_0L_0|p_4\rangle$ and also satisfies the property that any three point correlator in the first set involving one or two $\widetilde{p}_4$'s equals the correlator with $\widetilde{p}_4$ replaced by p_4 and φ_t replaced by $\varphi_a/\sqrt{2}$. (This can be checked by explicit computation.) Finally, any three point correlator involving two fields from the first set and the field d vanishes. This is trivially so for $\langle t, t, d\rangle$ and less trivially so for $\langle \varphi_t, \varphi_t, d\rangle$ and others.

It follows from the above argument that if we rewrite the relevant part of the string field $|\widetilde{\Phi}^{(4)}\rangle$ making use of the new basis

$$\widetilde{g}\,|p_4\rangle + t_2\,|\chi\rangle = \frac{1}{6}(t_2 - 3\widetilde{g})\,|\widetilde{p}_4\rangle + \frac{1}{6}\,(t_2 + 9\widetilde{g})\,|d\rangle\,, \tag{3.18}$$

Level	$\bar{t}_1$	$t_1^{(0)}$	$t_0^{(0)}$	$t_2^{(0)}$
$(1,2)$	0.4092	$\pm$ 0.21307	0.03336	—
$(2,4)$	0.4462	$\pm$ 0.29707	0.06964	—
$(3,6)$	0.4598	$\pm$ 0.31127	0.07693	—
$(4,8)$	0.4624	$\pm$ 0.33625	0.09425	-0.0102

Table 2: We show the variation of various quatities as a function of the level of the calculation for $R = \sqrt{1.1}$. Here $\bar{t}_1$ denotes the maximal value possible for the tachyon harmonic t_1. The next three columns give the values of the tachyon harmonics at the lump solution of the equations of motion. Note that as the level is increased, the vev of the nearly marginal tachyon harmonic t_1 increases.

the component field associated to $|d\rangle$ will not acquire an expectation value as it has no one point functions with other fields that do. In addition, the interchangeability of $(p_4, \varphi_a/\sqrt{2})$ and $(\widetilde{p}_4, \varphi_t)$ in the relevant computations indicates that the coefficient field of $|\widetilde{p}_4\rangle$ in $|\widetilde{\Phi}^{(4)}\rangle$ should be identified with the field g, — the coefficient of $|p_4\rangle$ in $|\Phi^{(4)}\rangle$. All in all we have that the interactions involving level four fields will agree upon the identifications

$$g = \frac{1}{6}(t_2 - 3\widetilde{g}) \qquad \text{and} \qquad 0 = \frac{1}{6}(t_2 + 9\widetilde{g})\,. \tag{3.19}$$

In view of this and previous identifications, our result for the equivalence of the Wilson and tachyon lump string field potentials (ommitting all common field variables that are simply identified) reads

$$\begin{aligned}\mathcal{V}\Big(a_s, \{s, r, \bar{r}, y\}, g\Big) = \widetilde{\mathcal{V}}\Big(t_1 = \sqrt{2}a_s, \{u_1, v_1, w_1, z_1\} = \sqrt{2}\{s, r, \bar{r}, y\}\,,\\ \widetilde{g} = -g/2, t_2 = 9g/2\,; R = 1\Big)\end{aligned} \tag{3.20}$$

This is the main result of this subsection. Integrating out all fields except t_1 or a_s yields the result quoted in (3.11).

3.4 Estimating the string field marginal parameter for the lump

Having noted at the end of subsection 3.1 that taking the limit $R \to 1$ at any fixed level does not provide an estimate for the vev of t_1 at the lump solution, we try now taking a fixed value of R near one, and examine how the vev of t_1 at the lump solution varies as we increase the level. We will take, somewhat arbitrarily, $R = \sqrt{1.1}$, a value of R reasonably close to one, but (hopefully) not so close that we would need a prohibitively high level computation to converge to the expected values of t_1.

Using the potential given in appendix B, and setting $R = \sqrt{1.1}$, we can calculate the value of the string field at the extremum of the potential representing the lump at different levels of approximation. In table 2 we have given the values $t_0^{(0)}$, $t_1^{(0)}$

JHEP10(2000)009

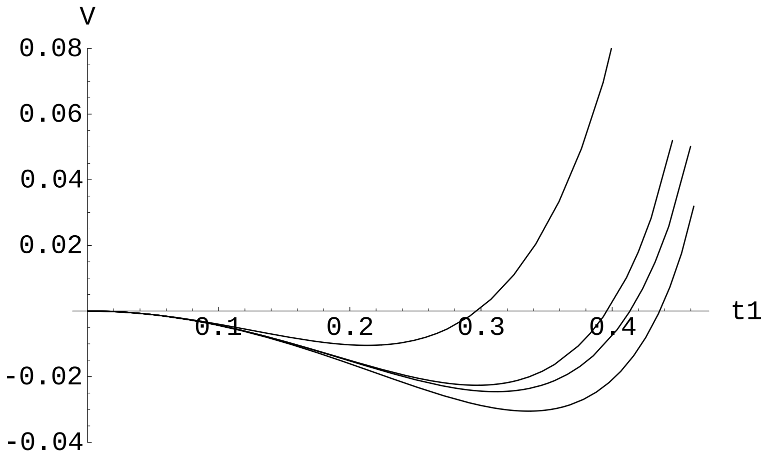

Figure 4: The level $(1,2)$, $(2,4)$, $(3,\ 6)$ and $(4,8)$ effective potentials $V \equiv 2\pi^2 \widetilde{\mathcal{V}}^M(t_1; R)$ for t_1 when $R = \sqrt{1.1}$. As the level is increased the potentials become deeper and the value of t_1 at the minimum larger.

and $t_2^{(0)}$ of the tachyon harmonics at this extremum. Further insight is obtained by consideration of the effective potentials $\widetilde{\mathcal{V}}^M(t_1; R = \sqrt{1.1})$ obtained at various approximation levels and shown in figure 4. Note that with increasing level, the potentials become increasingly deep and the minimum is attained for larger and larger values of t_1. Those values $t_1^{(0)}$ for t_1 are the ones given on the table. At level $(4,8)$, the value of t_1 at the minimum of the potential is $\pm.336$. Assuming that this is a good approximation to $t_1^{(0)}$ for $R = 1$, and using eq. (3.4), we see that the lump solution at $R = 1$ corresponds to $a_s = \pm.336/\sqrt{2} = \pm.238$. This gives

$$a_s \simeq \pm\, .238 \qquad \text{at} \qquad a_c = \pm\frac{1}{2\sqrt{2}}\,. \tag{3.21}$$

Due to the reasons mentioned at the end of subsection 3.1, however, this value of $t_1^{(0)}$ might not be a very accurate result, since even at level $(4,8)$ $\mathcal{V}^M(a_s) = \widetilde{\mathcal{V}}^M(\sqrt{2}a_s, R{=}1)$ receives an appreciable contribution, and hence causes a significant distortion of the potential at $R = \sqrt{1.1}$. Indeed, the pattern in table 2 suggests that at least at this radius, we are underestimating the value of $t_1^{(0)}$.

4. Matching CFT and SFT marginal parameters

In this section we shall try to interpret our results of section 2. In particular we shall be addressing the question: what does it mean to have a finite cut-off $\overline{a}_s$ on a_s beyond which the string field theory calculation of the effective potential breaks down?

Clearly the most important question here is: what gauge field vev does the point $a_s = \overline{a}_s$ correspond to? If it corresponds to infinite value of this gauge field then our results would imply that in string field theory a finite range of the string field covers the full range of values of the marginal deformation parameter in conformal field theory. On the other hand if $a_s = \overline{a}_s$ corresponds to a finite value of the gauge field vev, then it would mean that formulated around a given background, the string field theory only covers a finite subset of the full CFT moduli space.

In order to be able to resolve this issue, one needs to know the relationship between the string field theory parameter a_s and the gauge field vev a_c in the Born-Infeld action. In general the parameters a_s and a_c are related by a function:[8]

$$a_s = f(a_c)\,. \tag{4.1}$$

If $|\varphi\rangle$ denotes the normalized dimension one primary state representing the marginal direction, and φ denotes the corresponding vertex operator, then we take a_s to be the coefficient of the state $c_1|\varphi\rangle$ in the expansion of the string field, and a_c to be coefficient of $\int dx\varphi(x)$ to be added to the CFT action in order to construct the marginally deformed CFT. With this normalization convention, for small a_c, $a_s \simeq a_c$ [30]. This gives $f(a_c) \simeq a_c$ for small a_c.

Our interest lies in studying the behaviour of $f(a_c)$ for large a_c. In particular, we want to explore the possibility that $a_s = \overline{a}_s$ corresponds to $a_c = \infty$. It is easy to construct functions which approach a finite value $\overline{a}_s$ for large a_c. An example of a function of this type is:

$$f(a_c) = \overline{a}_s \tanh\left(\frac{a_c}{\overline{a}_s}\right). \tag{4.2}$$

This would have the requisite properties $f(a_c) \approx a_c$ for a_c small, and $f(\infty) = \overline{a}_s$. Using the level $(4,8)$ value of $\overline{a}_s$ $(= .331)$, eq. (4.2) predicts:

$$f\left(a_c = \frac{1}{2\sqrt{2}}\right) = .331 \tanh \frac{1}{.331 \times 2\sqrt{2}} = .261\,. \tag{4.3}$$

This is in fair agreement with eqs. (3.21). Indeed, as remarked below that equation, (3.21) is probably an underestimate of the actual value of a_s for $a_c = 1/2\sqrt{2}$.

Although this analysis seems to indicate that eq. (4.2) gives a fairly accurate description of the relationship between a_s and a_c, there is also counterevidence to this conjectured relationship. For this we again turn to the potential $\widetilde{\mathcal{V}}^M(t_1; R)$ in the tachyonic lump problem. According to the analysis of ref. [24], for $R > 1$ but close to 1, the effective potential is periodic in a_c with periodicity $1/\sqrt{2}$. This means that the potential should have an infinite number of oscillations in the range $0 \le a_c < \infty$, and in particular have a maximum at $a_c = 1/\sqrt{2}$. According to eq. (4.2) this corresponds to the point $a_s = .322$, i.e. $t_1 = \sqrt{2} \times .322 = .455$. Examining figure 4 we find no evidence for a maximum of $\widetilde{\mathcal{V}}^M(t_1; R)$ near $t_1 \sim .455$, nor any oscillation of the potential. This seems to indicate that eq. (4.2) does not quite represent the correct relation between a_s and a_c, and that $a_s = \overline{a}_s$ may correspond to a value of a_c below

[8] Some aspect of this relationship has been discussed recently in ref. [29].

$1/\sqrt{2}$. We should recall, however, that $\mathcal{V}^M(a_s) = \widetilde{\mathcal{V}}^M(\sqrt{2}a_s, R=1)$ computed at level $(4,8)$ has a large slope at $a_s = .322$ (see figure 2), and this might destroy a potential maximum in $\widetilde{\mathcal{V}}^M(\sqrt{2}a_s; R)$ at this point. Thus in absence of better numerical results, we are unable to decide whether $a_s = \overline{a}_s$ corresponds to infinite or finite a_c.

5. Concluding remarks

The finite range of definition of the effective potential for the string field marginal parameter a_s in the (marginal) M-branch arose because beyond some limiting value for a_s field equations for other string fields could not be solved at all or solutions would fail to be continuous functions of a_s. More concretely, we saw that at the critical value for a_s the M-branch and the V-branch, associated with the stable tachyonic vacuum merged.

Actually, even the tachyon effective potential appears to have a finite range of definition in the level expansion [11, 6] with the stable minimum well inside this range. In this case the limiting values appear as points beyond which some massive field equations either fail to have solutions or fail to give solutions that are continuous functions of the tachyon. The physical interpretation of this finite range is not clear; in particular, the lower limiting value is precisely in the direction where the effective potential of the tachyon is expected to be unbounded below. A complete understanding of the more familiar marginal case discussed in this paper might help interpret the finite range of the tachyon effective potential.

For the case of superstring field theory on a BPS D-brane, the non-BPS D-brane, or the D-brane anti-D-brane pair, if string field marginal parameters have finite ranges it will be through an effect technically similar to that of the tachyon effective potential. Given that in such superstring field theories the fields in the GSO odd sector acquire no expectation values, unfamiliar branches of solutions associated to massive fields in the GSO even sector would have to limit the domain of definition of the effective potential.

It certainly appears that more accurate calculations could give significant insight into the questions raised in this paper. A proper understanding of the description of marginal operators in both bosonic string theory and superstring theory promises to deepen considerably our understanding of the way non-perturbative physics is encoded in string field theory.

Acknowledgments

We are grateful to W. Taylor for detailed discussions on various aspects of this work and for comparing with us his unpublished results. We would also like to thank L. Rastelli for useful discussions. A.S. would like to thank the New High Energy Theory Center at Rutgers University, Center for Theoretical Physics at MIT and

Erwin Schroedinger Institute, Vienna for hospitality during the course of the work. The work of B.Z. was supported in part by DOE contract #DE-FC02-94ER40818.

A. SFT potential for study of Wilson line

In this appendix we shall derive the string field theory potential $\mathcal{V}(\Phi)$ (with normalization as defined in the text) relevant for studying the effective potential for constant gauge field configuration. The expansion of the string field has been described in section 2. We shall denote by $\mathcal{V}^{(M,N)}$ the level (M,N) approximation to $\mathcal{V}(\Phi)$, by $\mathcal{V}_{mm}$ the quadratic term in the potential for level m fields, and by $\mathcal{V}_{mnp}$ the cubic term in the potential coupling a level m, a level n and a level p field. As discussed in the text, using twist symmetry of the action, it is easy to see that the total level of all the fields entering a term in the potential must be even. We then have:

$$\begin{aligned}
\mathcal{V}^{(0,0)} &= \mathcal{V}_{00} + \mathcal{V}_{000} \\
\mathcal{V}^{(1,2)} &= \mathcal{V}^{(0,0)} + \mathcal{V}_{11} + \mathcal{V}_{011} \\
\mathcal{V}^{(2,4)} &= \mathcal{V}^{(1,2)} + \mathcal{V}_{22} + \mathcal{V}_{002} + \mathcal{V}_{112} + \mathcal{V}_{022} \\
\mathcal{V}^{(3,6)} &= \mathcal{V}^{(2,4)} + \mathcal{V}_{33} + \mathcal{V}_{013} + \mathcal{V}_{222} + \mathcal{V}_{033} + \mathcal{V}_{123} \\
\mathcal{V}^{(4,8)} &= \mathcal{V}^{(3,6)} + \mathcal{V}_{44} + \mathcal{V}_{004} + \mathcal{V}_{114} + \mathcal{V}_{024} + \mathcal{V}_{233} + \mathcal{V}_{224} + \mathcal{V}_{044} + \mathcal{V}_{134}
\end{aligned} \tag{A.1}$$

We let $K = 3\sqrt{3}/4$ as in the text. Explicit computation gives the following expressions for $\mathcal{V}_{mm}$ and $\mathcal{V}_{mnp}$:

$$\begin{aligned}
\mathcal{V}_{00} &= -\frac{1}{2}t_0^2 \\
\mathcal{V}_{11} &= 0 \\
\mathcal{V}_{22} &= -\frac{1}{2}u_0^2 + \frac{1}{4}v_0^2 + \frac{25}{4}w_0^2 \\
\mathcal{V}_{33} &= -s^2 + \frac{9}{2}r^2 + \frac{25}{2}\bar{r}^2 + 12y^2 + 12ry \\
\mathcal{V}_{44} &= \frac{15}{2}a^2 + \frac{375}{2}\bar{a}^2 + 9ab + \frac{27}{4}b^2 + 225\bar{a}\bar{b} + \frac{2475}{4}\bar{b}^2 + \frac{75}{8}\widehat{b}^2 + 3cd - \\
&\quad - \frac{3}{2}e^2 - \frac{3}{4}f^2 - \frac{75}{4}\bar{f}^2 + \frac{81}{4}g^2 \\
\mathcal{V}_{000} &= \frac{1}{3}K^3 t_0^3 \\
\mathcal{V}_{011} &= K t_0 a_s^2 \\
\mathcal{V}_{002} &= \frac{1}{32}K t_0^2 (22u_0 - 5v_0 - 125w_0) \\
\mathcal{V}_{112} &= \frac{1}{32}K^{-1} a_s^2 (22u_0 + 27v_0 - 125w_0) \\
\mathcal{V}_{013} &= \frac{1}{16}K^{-1} t_0 a_s \Big(22s - 21r - 125\bar{r} - 12y\Big) \\
\mathcal{V}_{022} &= \frac{1}{1024}K^{-1} t_0 (228u_0^2 - 220u_0v_0 + 537v_0^2 - 5500u_0w_0 + 1250v_0w_0 + 28425w_0^2)
\end{aligned}$$

$$
\begin{aligned}
\mathcal{V}_{004} \;=\; & \frac{1}{1024}K^{-1}t_0^2\Big(540a + 13500\bar{a} + 459b + 26475\bar{b} + 625\widehat{b} - \\
& \qquad - 320c + 960d + 228e - 110f - 2750\bar{f}\Big) \\
\mathcal{V}_{222} \;=\; & K\left\{\frac{1}{144}u_0^3 + \frac{8321}{93312}v_0^3 - \frac{219775}{10368}w_0^3 - \frac{95}{7776}u_0^2\left(v_0 + 25w_0\right) + \right. \\
& \left. + \frac{1969}{15552}u_0v_0^2 + \frac{104225}{15552}u_0w_0^2 - \frac{22375}{31104}v_0^2w_0 - \frac{47375}{31104}v_0w_0^2 + \frac{6875}{23328}u_0v_0w_0\right\} \\
\mathcal{V}_{033} \;=\; & \frac{1}{1728}K^{-1}t_0\Big(228s^2 - 924sr + 9657r^2 - 5500s\bar{r} + 5250r\bar{r} + 28425\bar{r}^2 - \\
& \qquad - 528sy + 31224ry + 3000\bar{r}y + 30864y^2\Big) \\
\mathcal{V}_{123} \;=\; & \frac{1}{864}K^{-1}a_s\Big(228u_0s + 594sv_0 - 462u_0r + 969v_0r - 2750sw_0 + 2625rw_0 - \\
& \qquad - 2750u_0\bar{r} - 3375v_0\bar{r} + 28425w_0\bar{r} - 264u_0y - 324v_0y + 1500w_0y\Big) \\
\mathcal{V}_{114} \;=\; & \frac{1}{1728}K^{-1}a_s^2\Big(-868a + 13500\bar{a} - 885b + 26475\bar{b} - 3375\widehat{b} - 320c + \\
& \qquad + 960d + 228e + 594f - 2750\bar{f} + 3072g\Big) \\
\mathcal{V}_{024} \;=\; & \frac{1}{16384}K^{-3}t_0\bigg(u_0\Big(11880a + 297000\bar{a} + 10098b + 582450\bar{b} + 13750\widehat{b} - 9600c + \\
& \qquad + 28800d + 30616e - 1140f - 28500\bar{f}\Big) + \\
& \qquad + v_0\Big(7540a - 67500\bar{a} - 23799b - 132375\bar{b} - 67125\widehat{b} + 1600c - \\
& \qquad - 4800d - 1140e + 11814f + 13750\bar{f}\Big) + \\
& \qquad + w_0\Big(-67500a - 1431500\bar{a} - 57375b - 6918975\bar{b} - 142125\widehat{b} + \\
& \qquad + 40000c - 120000d - 28500e + 13750f + 625350\bar{f}\Big)\bigg) \\
\mathcal{V}_{233} \;=\; & \frac{1}{18432}K^{-3}\Big(648u_0s^2 + 2052s^2v_0 - 3192u_0sr + 14212sv_0r + 70818u_0r^2 + \\
& +16257v_0r^2 - 9500s^2w_0 + 38500srw_0 - 402375r^2w_0 - 19000u_0s\bar{r} - \\
& -49500sv_0\bar{r} + 38500u_0r\bar{r} - 80750v_0r\bar{r} + 416900sw_0\bar{r} - \\
& -397950rw_0\bar{r} + 208450u_0\bar{r}^2 + 255825v_0\bar{r}^2 - 1977975w_0\bar{r}^2 - \\
& -1824u_0sy - 4752sv_0y + 228976u_0ry - 58952v_0ry + \\
& +22000sw_0y - 1301000rw_0y + 22000u_0\bar{r}y + 27000v_0\bar{r}y - \\
& -227400w_0\bar{r}y + 226336u_0y^2 - 49904v_0y^2 - 1286000w_0y^2\Big) \\
\mathcal{V}_{224} \;=\; & \frac{1}{1048576}K^{-5}\bigg(u_0^2\Big(123120a + 3078000\bar{a} + 104652b + 60363000\bar{b} + 142500\widehat{b} - \\
& -103680c + 311040d + 1997584e - 9720f - 243000\bar{f}\Big) + \\
& +u_0v_0\Big(331760a - 2970000\bar{a} - 1047156b - 5824500\bar{b} - 2953500\widehat{b} +
\end{aligned}
$$

$$
\begin{aligned}
&+96000c-288000d-306160e+244872f+285000\bar{f}\Big)+\\
&+v_0^2\Big(-1254212a+7249500\bar{a}+658131b+14217075\bar{b}-3120375\widehat{b}-\\
&-171840c+515520d+122436e+549186f-1476750\bar{f}\Big)+\\
&+u_0w_0\Big(-2970000a-62986000\bar{a}-2524500b-304434900\bar{b}-6253500\widehat{b}+\\
&+2400000c-7200000d-7654000e+285000f+12961800\bar{f}\Big)+\\
&+v_0w_0\Big(-1885000a+14315000\bar{a}+5949750b+691189750\bar{b}+305284500\widehat{b}-\\
&-400000c+1200000d+285000e-2953500f-6253500\bar{f}\Big)+\\
&+w_0^2\Big(15349500a+283692700\bar{a}+130470750b+1777831875\bar{b}+296696250\widehat{b}-\\
&-9096000c+27288000d+6480900e-3126750f-130546350\bar{f}\Big)\Big)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{V}_{044}=\frac{1}{1048576}K^{-5}t_0\Big(&18846480a^2+14580000a\bar{a}+646122000\bar{a}^2+21655656ab+\\
&+12393000\bar{a}b+10551033b^2+28593000a\bar{b}+1120943400\bar{a}\bar{b}+\\
&+24304050b\bar{b}+22693352250\bar{b}^2-1885000a\widehat{b}+14315000\bar{a}\widehat{b}+\\
&+5949750b\widehat{b}+69189750\bar{b}\widehat{b}+152642250\widehat{b}^2-345600ac-8640000\bar{a}c-\\
&-293760bc-16944000\bar{b}c-400000\widehat{b}c+76800c^2+1036800ad+\\
&+25920000\bar{a}d+881280bd+50832000\bar{b}d+1200000\widehat{b}d+1636352cd+\\
&+691200d^2+246240ae+6156000\bar{a}e+209304be+12072600\bar{b}e+\\
&+285000\widehat{b}e+1021440ce-3064320de-1754352e^2+331760af-\\
&-2970000\bar{a}f-1047156bf-5824500\bar{b}f-2953500\widehat{b}f+96000cf-\\
&-288000df-306160ef+122436f^2-2970000a\bar{f}-62986000\bar{a}\bar{f}-\\
&-2524500b\bar{f}-304434900\bar{b}\bar{f}-6253500\widehat{b}\bar{f}+2400000c\bar{f}-\\
&-7200000d\bar{f}-7654000e\bar{f}+285000f\bar{f}+6480900\bar{f}^2\Big)+\frac{27}{2}K^{-5}t_0g^2
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{V}_{134}=\frac{1}{16384}K^{-5}a_s\Big(&r\Big(196404a-283500\bar{a}+298137b-555975\bar{b}-121125\widehat{b}+\\
&+6720c-20160d-4788e+21318f+57750\bar{f}+132096g\Big)+\\
&+\bar{r}\Big(108500a-1431500\bar{a}+110625b-6918975\bar{b}+767475\widehat{b}+\\
&+40000c-120000d-28500e-74250f+625350\bar{f}-384000g\Big)+\\
&+s\Big(-19096a+297000\bar{a}-19470b+582450\bar{b}-74250\widehat{b}-\\
&-9600c+28800d+30616e+6156f-28500\bar{f}+67584g\Big)+\\
&+y\Big(305328a-162000\bar{a}+502140b-317700\bar{b}+40500\widehat{b}+\\
&+3840c-11520d-2736e-7128f+33000\bar{f}+749568g\Big)\Big)\,. \qquad \text{(A.2)}
\end{aligned}
$$

B. SFT potential for study of lump solution on a circle

In this appendix we shall derive the string field theory potential $\widetilde{\mathcal{V}}(\Phi)$ relevant for studying the formation of the lump solution on a circle of radius R. The expansion of the string field has been described in section 3. As mentioned there, since we are interested in studying this phenomenon near $R=1$, in counting level of a field we shall pretend as if R has already been set to 1, although in the expression for the potential we shall keep the complete R dependence. We shall denote by $\widetilde{\mathcal{V}}^{(M,N)}$ the level (M,N) approximation to $\widetilde{\mathcal{V}}(\Phi)$, by $\widetilde{\mathcal{V}}_{mm}$ the quadratic term in the potential for level m fields, and by $\widetilde{\mathcal{V}}_{mnp}$ the cubic term in the potential coupling a level m, a level n and a level p field. As discussed in the text, using momentum conservation it is easy to show that with this definition of level, the total level of all the fields entering a term in the potential must be even. We then have:

$$\begin{aligned}
\widetilde{\mathcal{V}}^{(0,0)} &= \widetilde{\mathcal{V}}_{00} + \widetilde{\mathcal{V}}_{000} \\
\widetilde{\mathcal{V}}^{(1,2)} &= \widetilde{\mathcal{V}}^{(0,0)} + \widetilde{\mathcal{V}}_{11} + \widetilde{\mathcal{V}}_{011} \\
\widetilde{\mathcal{V}}^{(2,4)} &= \widetilde{\mathcal{V}}^{(1,2)} + \widetilde{\mathcal{V}}_{22} + \widetilde{\mathcal{V}}_{002} + \widetilde{\mathcal{V}}_{112} + \widetilde{\mathcal{V}}_{022} \\
\widetilde{\mathcal{V}}^{(3,6)} &= \widetilde{\mathcal{V}}^{(2,4)} + \widetilde{\mathcal{V}}_{33} + \widetilde{\mathcal{V}}_{013} + \widetilde{\mathcal{V}}_{222} + \widetilde{\mathcal{V}}_{033} + \widetilde{\mathcal{V}}_{123} \\
\widetilde{\mathcal{V}}^{(4,8)} &= \widetilde{\mathcal{V}}^{(3,6)} + \widetilde{\mathcal{V}}_{44} + \widetilde{\mathcal{V}}_{004} + \widetilde{\mathcal{V}}_{114} + \widetilde{\mathcal{V}}_{024} + \widetilde{\mathcal{V}}_{233} + \widetilde{\mathcal{V}}_{224} + \widetilde{\mathcal{V}}_{044} + \widetilde{\mathcal{V}}_{134}\,.
\end{aligned} \tag{B.1}$$

Explicit computation gives the following expressions for $\widetilde{\mathcal{V}}_{mm}$ and $\widetilde{\mathcal{V}}_{mnp}$:

$$\begin{aligned}
\widetilde{\mathcal{V}}_{00} &= -\frac{1}{2}t_0^2 \\
\widetilde{\mathcal{V}}_{11} &= -\frac{1}{4}(1-R^{-2})t_1^2 \\
\widetilde{\mathcal{V}}_{22} &= -\frac{1}{2}u_0^2 + \frac{1}{4}v_0^2 + \frac{25}{4}w_0^2 \\
\widetilde{\mathcal{V}}_{33} &= -\frac{1}{4}u_1^2(1+R^{-2}) + \frac{1}{8}v_1^2(1+R^{-2})(1+8R^{-2}) + \frac{25}{8}w_1^2(1+R^{-2}) + \\
&\quad +3R^{-2}v_1z_1(1+R^{-2}) + R^{-2}z_1^2(1+R^{-2})(1+2R^{-2}) \\
\widetilde{\mathcal{V}}_{44} &= \frac{15}{2}a^2 + \frac{375}{2}\bar{a}^2 + 9ab + \frac{27}{4}b^2 + 225\bar{a}\bar{b} + \frac{2475}{4}\bar{b}^2 + \frac{75}{8}\widehat{b}^2 + 3cd - \\
&\quad -\frac{3}{2}e^2 - \frac{3}{4}f^2 - \frac{75}{4}\bar{f}^2 + \frac{81}{4} - \widetilde{g}^2\frac{1}{4}(1-4R^{-2})t_2^2 \\
\widetilde{\mathcal{V}}_{000} &= \frac{1}{3}K^3t_0^3 \\
\widetilde{\mathcal{V}}_{011} &= \frac{1}{2}K^{3-2/R^2}t_0t_1^2 \\
\widetilde{\mathcal{V}}_{002} &= \frac{1}{32}Kt_0^2\,(22u_0 - 5v_0 - 125w_0) \\
\widetilde{\mathcal{V}}_{112} &= K^{1-2/R^2}t_1^2\left(\frac{11}{32}u_0 + \frac{1}{2}\Big(-\frac{5}{32}+R^{-2}\Big)v_0 - \frac{125}{64}w_0\right)
\end{aligned}$$

JHEP10(2000)009

$$
\begin{aligned}
\widetilde{\mathcal{V}}_{013} &= \frac{1}{32}K^{1-2/R^2}t_0t_1\Big(22u_1-(5+16R^{-2})v_1-125w_1+R^{-2}(-44+32R^{-2})z_1\Big)\\
\widetilde{\mathcal{V}}_{022} &= \frac{1}{1024}K^{-1}t_0\left(228u_0^2-220u_0v_0+537v_0^2-5500u_0w_0+1250v_0w_0+28425w_0^2\right)\\
\widetilde{\mathcal{V}}_{004} &= \frac{1}{1024}K^{-1}t_0^2\Big(540a+13500\bar{a}+459b+26475\bar{b}+625\widehat{b}-\\
&\qquad\qquad -320c+960d+228e-110f-2750\bar{f}\Big)\\
\widetilde{\mathcal{V}}_{222} &= K\Bigg\{\frac{1}{144}u_0^3+\frac{8321}{93312}v_0^3-\frac{219775}{10368}w_0^3-\frac{95}{7776}u_0^2(v_0+25w_0)+\frac{1969}{15552}u_0v_0^2+\\
&\quad +\frac{104225}{15552}u_0w_0^2-\frac{22375}{31104}v_0^2w_0-\frac{47375}{31104}v_0w_0^2+\frac{6875}{23328}u_0v_0w_0\Bigg\}\\
\widetilde{\mathcal{V}}_{033} &= K^{1-2/R^2}\Bigg\{\frac{19}{288}t_0u_1^2+\frac{1}{3456}\left(537+\frac{8864}{R^2}+\frac{256}{R^4}\right)t_0v_1^2+\frac{28425}{3456}t_0w_1^2-\\
&\quad -\frac{11}{864}t_0u_1\left(\left(5+\frac{16}{R^2}\right)v_1+125w_1\right)+\frac{125}{1728}\left(5+\frac{16}{R^2}\right)t_0v_1w_1+\\
&\quad +\frac{11}{864}\frac{1}{R^2}\left(-44+\frac{32}{R^2}\right)t_0u_1z_1+\frac{1}{432}\left(\frac{2359}{R^2}+\frac{1672}{R^4}-\frac{128}{R^6}\right)t_0v_1z_1+\\
&\quad +\frac{125}{432}\frac{1}{R^2}\left(11-\frac{8}{R^2}\right)t_0w_1z_1+\frac{1}{216}\frac{1}{R^2}\left(384+\frac{1145}{R^2}+\frac{336}{R^4}+\frac{64}{R^6}\right)t_0z_1^2\\
\widetilde{\mathcal{V}}_{123} &= K^{1-2/R^2}\Bigg\{\frac{19}{144}t_1u_0u_1-\frac{11}{864}t_1u_0\left(\left(5+\frac{16}{R^2}\right)v_1+125w_1\right)+\\
&\quad +\frac{1}{864}\frac{1}{R^2}\left(11\left(-44+\frac{32}{R^2}\right)t_1u_0z_1+\left(2158-\frac{2832}{R^2}+\frac{512}{R^4}\right)t_1v_0z_1\right)-\\
&\quad -\frac{11}{864}\left(5-\frac{32}{R^2}\right)t_1v_0u_1+\frac{1}{1728}\left(537+\frac{944}{R^2}-\frac{512}{R^4}\right)t_1v_0v_1+\\
&\quad +\frac{25}{1728}\left(25-\frac{160}{R^2}\right)t_1v_0w_1-\frac{1375}{864}t_1w_0u_1+\\
&\quad +\frac{25}{1728}\left(25+\frac{80}{R^2}\right)t_1w_0v_1+\frac{28425}{1728}t_1w_0w_1+\frac{125}{432}\frac{1}{R^2}\left(11-\frac{8}{R^2}\right)t_1w_0z_1\Bigg\}\\
\widetilde{\mathcal{V}}_{114} &= K^{1-2/R^2}t_1^2\Bigg\{\Big(\frac{5}{32}-\frac{11}{27R^2}\Big)a+\frac{125}{32}\bar{a}+\Big(\frac{17}{128}-\frac{37}{54R^2}+\frac{8}{27R^4}\Big)b+\\
&\qquad +\frac{8825}{1152}\bar{b}+\Big(\frac{625}{3456}-\frac{125}{108R^2}\Big)\widehat{b}-\frac{5}{54}c+\frac{5}{18}d+\frac{19}{288}e+\\
&\qquad +\Big(-\frac{55}{1728}+\frac{11}{54R^2}\Big)f-\frac{1375}{1728}\bar{f}\Bigg\}+\\
&\qquad +\frac{1}{4}K^{3-6/R^2}t_1^2t_2+\frac{1}{4}K^{-1-2/R^2}R^{-2}(1-4R^{-2})t_1^2\widetilde{g}\\
\widetilde{\mathcal{V}}_{024} &= \frac{1}{16384}K^{-3}t_0\Big(u_0(11880a+297000\bar{a}+10098b+582450\bar{b}+13750\widehat{b}-\\
&\qquad\qquad -9600c+28800d+30616e-1140f-28500\bar{f})+\\
&\qquad\quad +v_0(7540a-67500\bar{a}-23799b-132375\bar{b}-67125\widehat{b}+
\end{aligned}
$$

$$
\begin{aligned}
&+1600c-4800d-1140e+11814f+13750\bar{f})+\\
&+w_0(-67500a-1431500\bar{a}-57375b-6918975\bar{b}-142125\widehat{b}+\\
&+40000c-120000d-28500e+13750f+625350\bar{f})\Big)
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{\mathcal{V}}_{233} = \frac{1}{65536}K^{-3-2/R^2}\Big(&1944u_0u_1^2+(7296R^{-2}-1140)v_0u_1^2-(7296R^{-2}+2280)u_0u_1v_1+\\
&+(-22528R^{-4}+41536R^{-2}+23628)v_0u_1v_1+\\
&+(5632R^{-4}+195008R^{-2}+11814)u_0v_1^2+\\
&+(8192R^{-6}+249600R^{-4}-233984R^{-2}+24963)v_0v_1^2-28500w_0u_1^2+\\
&+(88000R^{-2}+27500)w_0u_1v_1+(-32000R^{-4}-1108000R^{-2}-67125)w_0v_1^2-\\
&-57000u_0u_1w_1+(-176000R^{-2}+27500)v_0u_1w_1+\\
&+(88000R^{-2}+27500)u_0v_1w_1+\\
&+(128000R^{-4}-236000R^{-2}-134250)v_0v_1w_1+1250700w_0u_1w_1+\\
&+(-909600R^{-2}-284250)w_0v_1w_1+625350u_0w_1^2+\\
&+(909600R^{-2}-142125)v_0w_1^2-\\
&-5933925w_0w_1^2+(14592R^{-4}-20064R^{-2})u_0u_1z_1+\\
&+(45056R^{-6}-249216R^{-4}+189904R^{-2})v_0u_1z_1+\\
&+(-22528R^{-6}+294272R^{-4}+415184R^{-2})u_0v_1z_1+\\
&+(-32768R^{-8}+629760R^{-6}-790080R^{-4}+16232R^{-2})v_0v_1z_1+\\
&+(-176000R^{-4}+242000R^{-2})w_0u_1z_1+\\
&+(128000R^{-6}-1672000R^{-4}-2359000R^{-2})w_0v_1z_1+\\
&+(-176000R^{-4}+242000R^{-2})u_0w_1z_1+\\
&+(-256000R^{-6}+1416000R^{-4}-1079000R^{-2})v_0w_1z_1+\\
&+(1819200R^{-4}-2501400R^{-2})w_0w_1z_1+\\
&+(22528R^{-8}+118272R^{-6}+403040R^{-4}+135168R^{-2})u_0z_1^2+\\
&+(32768R^{-10}-95232R^{-8}+395520R^{-6}-517584R^{-4}+34816R^{-2})v_0z_1^2+\\
&+(-128000R^{-8}-672000R^{-6}-2290000R^{-4}-768000R^{-2})w_0z_1^2\Big)
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{\mathcal{V}}_{224} = \frac{1}{1048576}K^{-5}\Big(&u_0^2\Big(123120a+3078000\bar{a}+104652b+6036300\bar{b}+142500\widehat{b}-\\
&-103680c+311040d+1997584e-9720f-243000\bar{f}\Big)+\\
&+u_0v_0\Big(331760a-2970000\bar{a}-1047156b-5824500\bar{b}-2953500\widehat{b}+\\
&+96000c-288000d-306160e+244872f+285000\bar{f}\Big)+\\
&+v_0^2\Big(-1254212a+7249500\bar{a}+658131b+14217075\bar{b}-3120375\widehat{b}-\\
&-171840c+515520d+122436e+549186f-1476750\bar{f}\Big)+\\
&+u_0w_0\Big(-2970000a-62986000\bar{a}-2524500b-304434900\bar{b}-6253500\widehat{b}+
\end{aligned}
$$

$$
\begin{aligned}
&+2400000c - 7200000d - 7654000e + 285000f + 12961800\bar{f}\Big) + \\
&+v_0 w_0\Big(-1885000a + 14315000\bar{a} + 5949750b + 691189750\bar{b} + 305284450\widehat{b} - \\
&-400000c + 1200000d + 285000e - 2953500f - 6253500\bar{f}\Big) + \\
&+w_0^2\Big(15349500a + 283692700\bar{a} + 130470750b + 17778318750\bar{b} + 296696250\widehat{b} - \\
&-9096000c + 27288000d + 6480900e - 3126750f - 130546350\bar{f}\Big)\Big)
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{\mathcal{V}}_{044} = {} & \frac{1}{1048576}K^{-5}t_0\Big(18846480a^2 + 14580000a\bar{a} + 646122000\bar{a}^2 + 21655656ab + \\
&+12393000\bar{a}b + 10551033b^2 + 28593000a\bar{b} + 1120943400\bar{a}\bar{b} + \\
&+24304050b\bar{b} + 22693352250\bar{b}^2 - 1885000a\widehat{b} + 14315000\bar{a}\widehat{b} + \\
&+5949750b\widehat{b} + 691189750\bar{b}\widehat{b} + 152642250\widehat{b}^2 - 345600ac - 8640000\bar{a}c - \\
&-293760bc - 16944000\bar{b}c - 400000\widehat{b}c + 76800c^2 + 1036800ad + \\
&+25920000\bar{a}d + 881280bd + 50832000\bar{b}d + 1200000\widehat{b}d + 1636352cd + \\
&+691200d^2 + 246240ae + 6156000\bar{a}e + 209304be + 12072600\bar{b}e + \\
&+285000\widehat{b}e + 1021440ce - 3064320de - 1754352e^2 + 331760af - \\
&-2970000\bar{a}f - 1047156bf - 5824500\bar{b}f - 2953500\widehat{b}f + 96000cf - \\
&-288000df - 306160ef + 122436f^2 - 2970000a\bar{f} - 62986000\bar{a}\bar{f} - \\
&-2524500b\bar{f} - 304434900\bar{b}\bar{f} - 6253500\widehat{b}\bar{f} + 2400000c\bar{f} - \\
&-7200000d\bar{f} - 7654000e\bar{f} + 285000f\bar{f} + 6480900\bar{f}^2\Big) + \\
&+\frac{27}{2}K^{-5}t_0\widetilde{g}^2 + \frac{1}{2}K^{3-8/R^2}t_0 t_2^2
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{\mathcal{V}}_{134} = {} & \frac{1}{32768}K^{-3-2/R^2}t_1\Big((-30976R^{-2} + 11880)au_1 + 297000\bar{a}u_1 + \\
&+(22528R^{-4} - 52096R^{-2} + 10098)bu_1 + 582450\bar{b}u_1 + \\
&+(-88000R^{-2} + 13750)\widehat{b}u_1 - 9600cu_1 + 28800du_1 + 30616eu_1 + \\
&+(7296R^{-2} - 1140)fu_1 - 28500\bar{f}u_1 + \\
&+(22528R^{-4} + 166336R^{-2} + 7540)av_1 + (-216000R^{-2} - 67500)\bar{a}v_1 + \\
&+(-16384R^{-6} + 98304R^{-4} + 240016R^{-2} - 23799)bv_1 + \\
&+(-423600R^{-2} - 132375)\bar{b}v_1 + (64000R^{-4} - 118000R^{-2} - 67125)\widehat{b}v_1 + \\
&+(5120R^{-2} + 1600)cv_1 + (-15360R^{-2} - 4800)dv_1 \\
&+(-3648R^{-2} - 1140)ev_1 + (-11264R^{-4} + 20768R^{-2} + 11814)fv_1 + \\
&+(44000R^{-2} + 13750)\bar{f}v_1 + (176000R^{-2} - 67500)aw_1 - 1431500\bar{a}w_1 + \\
&+(-128000R^{-4} + 296000R^{-2} - 57375)bw_1 - 6918975\bar{b}w_1 + \\
&+(909600R^{-2} - 142125)\widehat{b}w_1 + 40000cw_1 - 120000dw_1 - 28500ew_1 + \\
&+(-88000R^{-2} + 13750)fw_1 + 625350\bar{f}w_1 + \\
&+(-45056R^{-6} + 193920R^{-4} + 156464R^{-2})az_1 + \\
&+(432000R^{-4} - 594000R^{-2})\bar{a}z_1 +
\end{aligned}
$$

$$
\begin{aligned}
&+(32768R^{-8} - 382976R^{-6} + 880736R^{-4} - 28388R^{-2})bz_1 + \\
&+(847200R^{-4} - 1164900R^{-2})\bar{b}z_1 + \\
&+(-28000R^{-6} + 708000R^{-4} - 539500R^{-2})\widehat{b}z_1 - \\
&-(10240R^{-4} - 14080R^{-2})cz_1 + (30720R^{-4} - 42240R^{-2})dz_1 + \\
&+(7296R^{-4} - 10032R^{-2})ez_1 + \\
&+(22528R^{-6} - 124608R^{-4} + 94952R^{-2})fz_1 + \\
&+(-88000R^{-4} + 121000R^{-2})\bar{f}z_1\Big) + \\
&+\frac{1}{64}K^{-3-2/R^2}t_1\widetilde{g}R^{-4}(-2+R)(2+R)\Big(22u_1 + (-16R^{-2} + 59)v_1 - 125w_1 + \\
&\qquad + (32R^{-4} - 300R^{-2} + 512)z_1\Big) + \\
&+\frac{1}{64}K^{1-6/R^2}t_1t_2\Big(22u_1 + (48R^{-2} - 5)v_1 - 125w_1 + (288R^{-4} - 44R^{-2})z_1\Big) \qquad \text{(B.2)}
\end{aligned}
$$

References

[1] E. Witten, *Noncommutative geometry and string field theory*, *Nucl. Phys.* **B 268** (1986) 253.

[2] N. Berkovits, *Superpoincare invariant superstring field theory*, *Nucl. Phys.* **B 450** (1995) 90 [`hep-th/9503099`].

[3] N. Berkovits and C.T. Echevarria, *Four-point amplitude from open superstring field theory*, *Phys. Lett.* **B 478** (2000) 343 [`hep-th/9912120`];
N. Berkovits, *A new approach to superstring field theory*, *Fortsch. Phys.* **48** (2000) 31 [`hep-th/9912121`].

[4] A. Sen and B. Zwiebach, *Tachyon condensation in string field theory*, *J. High Energy Phys.* **03** (2000) 002 [`hep-th/9912249`].

[5] W. Taylor, *D-brane effective field theory from string field theory*, *Nucl. Phys.* **B 585** (2000) 171 [`hep-th/0001201`].

[6] N. Moeller and W. Taylor, *Level truncation and the tachyon in open bosonic string field theory*, *Nucl. Phys.* **B 583** (2000) 105 [`hep-th/0002237`].

[7] N. Berkovits, *The tachyon potential in open Neveu-Schwarz string field theory*, *J. High Energy Phys.* **04** (2000) 022 [`hep-th/0001084`].

[8] N. Berkovits, A. Sen and B. Zwiebach, *Tachyon condensation in superstring field theory*, *Nucl. Phys.* **B 587** (2000) 147 [`hep-th/0002211`].

[9] P.-J.D. Smet and J. Raeymaekers, *Level four approximation to the tachyon potential in superstring field theory*, *J. High Energy Phys.* **05** (2000) 051 [`hep-th/0003220`].

[10] A. Iqbal and A. Naqvi, *Tachyon condensation on a non-BPS D-brane*, hep-th/0004015.

[11] V.A. Kostelecky and S. Samuel, *The static tachyon potential in the open bosonic string theory*, *Phys. Lett.* **B 207** (1988) 169.

[12] J.A. Harvey and P. Kraus, *D-branes as unstable lumps in bosonic open string field theory*, *J. High Energy Phys.* **04** (2000) 012 [hep-th/0002117].

[13] R. de Mello Koch, A. Jevicki, M. Mihailescu and R. Tatar, *Lumps and p-branes in open string field theory*, *Phys. Lett.* **B 482** (2000) 249 [hep-th/0003031].

[14] N. Moeller, A. Sen and B. Zwiebach, *D-branes as tachyon lumps in string field theory*, *J. High Energy Phys.* **08** (2000) 039 [hep-th/0005036].

[15] J.A. Harvey, D. Kutasov and E.J. Martinec, *On the relevance of tachyons*, hep-th/0003101.

[16] P. Fendley, H. Saleur and N.P. Warner, *Exact solution of a massless scalar field with a relevant boundary interaction*, *Nucl. Phys.* **B 430** (1994) 577 [hep-th/9406125].

[17] R. Gopakumar, S. Minwalla and A. Strominger, *Noncommutative solitons*, *J. High Energy Phys.* **05** (2000) 020 [hep-th/0003160].

[18] K. Dasgupta, S. Mukhi and G. Rajesh, *Noncommutative tachyons*, *J. High Energy Phys.* **06** (2000) 022 [hep-th/0005006].

[19] J.A. Harvey, P. Kraus, F. Larsen and E.J. Martinec, *D-branes and strings as non-commutative solitons*, *J. High Energy Phys.* **07** (2000) 042 [hep-th/0005031].

[20] D.P. Jatkar, G. Mandal and S.R. Wadia, *Nielsen-Olesen vortices in noncommutative abelian Higgs model*, *J. High Energy Phys.* **09** (2000) 018 [hep-th/0007078].

[21] E. Witten, *Noncommutative tachyons and string field theory*, hep-th/0006071.

[22] A. Sen and B. Zwiebach, *A proof of local background independence of classical closed string field theory*, *Nucl. Phys.* **B 414** (1994) 649 [hep-th/9307088].

[23] A. Recknagel and V. Schomerus, *Boundary deformation theory and moduli spaces of D-branes*, *Nucl. Phys.* **B 545** (1999) 233 [hep-th/9811237];
C.G. Callan, I.R. Klebanov, A.W.W. Ludwig and J.M. Maldacena, *Exact solution of a boundary conformal field theory*, *Nucl. Phys.* **B 422** (1994) 417 [hep-th/9402113];
J. Polchinski and L. Thorlacius, *Free fermion representation of a boundary conformal field theory*, *Phys. Rev.* **D 50** (1994) 622 [hep-th/9404008].

[24] A. Sen, *Descent relations among bosonic D-branes*, *Int. J. Mod. Phys.* **A 14** (1999) 4061 [hep-th/9902105].

[25] L. Rastelli and B. Zwiebach, *Tachyon potentials, star products and universality*, hep-th/0006240.

[26] W. Taylor, in preparation.

[27] M.R. Gaberdiel and B. Zwiebach, *Tensor constructions of open string theories, 1. Foundations*, *Nucl. Phys.* **B 505** (1997) 569 [hep-th/9705038].

[28] A. Sen, *Universality of the tachyon potential*, *J. High Energy Phys.* **12** (1999) 027 [hep-th/9911116].

[29] J.R. David, U(1) *gauge invariance from open string field theory*, *J. High Energy Phys.* **10** (2000) 017 [hep-th/0005085].

[30] A. Sen, *On the background independence of string field theory*, *Nucl. Phys.* **B 345** (1990) 551.